Blind Estimation of Transmit Power in Wireless Networks

Murtaza Zafer (IBM Research), Bongjun Ko (IBM Research), Chatschik Bisdikian (IBM Research) and Ivan Ho (Imperial College, UK)
Transmit-power Estimation: Problem Synopsis

- Node T is a wireless transmitter with Tx power = \( P \) (unknown), at a position (\( x, y \)) (unknown)

- Nodes \( \{m_1, ..., m_N\} \) are monitors that measure received power \( \{p_i\} \)

- **Goal** – given \( \{p_i\} \) and \( \{(x_i, y_i)\} \) (monitor locations), estimate unknown \( P \) and (\( x, y \)).

- **Difficulty**: “Blind” estimation – no prior knowledge (statistical or otherwise) of the location or transmit power of the transmitter.
Motivation

- **Applications**
  - Signal jamming attack detection in MANET.
  - Node mis-configuration detection.
  - Primary user detection in cognitive radio networks.
  - Event intensity detection in sensor network.
  - Power-aware radio resource control with unknown transmit power.
  - Location identification of wireless users.
Wireless signal attenuation

\[ P = \text{transmission power} \]
\[ P_r = \text{received power} \]
\[ d = \text{distance between the transmitter and receiver} \]
\[ \alpha = \text{attenuation factor, (} \alpha > 1 \text{)} \]
\[ k = \text{normalizing constant} \]

**Deterministic model**

\[ P_r = \frac{kP}{d^\alpha} \]

**Stochastic model**

\[ P_i = H_i \frac{kP}{d_i^\alpha} \]

\[ H_i = e^{W_i}; \text{ lognormal r.v.} \]
Estimation under deterministic model

Deterministic propagation model:

\[ P_r = kP/d^\alpha \]

Single monitor measurement

best estimate of transmit power:

\[ P^* = P_1 \]
Two monitors

Locus of transmitter: using

\[
\frac{d_1}{d_2} = \left( \frac{P_{i2}}{P_{i1}} \right)^\frac{1}{\alpha} = c_1
\]

Lower bound of \( P^* \):

By the triangular inequality: \( d_1 + d_2 \geq d_{12} \)

\[
P_i^* = \left( \frac{d_{12}}{\left( k/P_i \right)^\frac{1}{\alpha} + \left( k/P_{i2} \right)^\frac{1}{\alpha}} \right)^\alpha
\]
Multiple monitors

Multiple monitor scenario

\[
\frac{d_1}{d_2} = \left( \frac{P_2}{P_1} \right)^{1/\alpha} (= c_1); \quad \frac{d_2}{d_3} = \left( \frac{P_3}{P_2} \right)^{1/\alpha} (= c_2); \quad \cdots \quad \frac{d_{N-1}}{d_N} = \left( \frac{P_N}{P_{N-1}} \right)^{1/\alpha} (= c_{N-1})
\]

• With multiple monitors – diversity in measurements
• System of equations with unknowns \((x, y, P)\)
• \textit{We should be able to solve these equations to obtain exact } P ?

\textbf{Answer: Yes and No !!!}
Theorem I: There is a unique solution \((P^*, x^*, y^*)\) except when the monitors are placed on an arc of a circle.

Proof:

- A location \((x, y)\) is a solution if and only if it satisfies \(d_1/d_2 = c_1, \ldots, d_{N-1}/d_N = c_{N-1}\)
- The actual location \((x_r, y_r)\) is one solution; thus \(d_{r,1}/d_{r,2} = c_1, \ldots, d_{r,N-1}/d_{r,N} = c_{N-1}\)
- There exists another solution at \((x, y)\) if and only if, \(d_r / d_r,2 = d_1/d_2, \ldots;\) equivalently,
  \[
  \frac{d_i}{d_{r,i}} = \beta, \text{ (a constant), } \forall i = 1, \ldots, N
  \]
Deterministic model

Multiple monitor scenario

**Corollary 1:** Two monitors always has multiple solutions
Deterministic model

Multiple monitor scenario

**Corollary 1:** Two monitors always has multiple solutions

**Corollary 2:** Three monitors always has multiple solutions

*In general, for any regular polygon placement of monitors the transmission power cannot be uniquely determined!*

For all non-circular placement of monitors, transmission power can be uniquely determined.
Stochastic attenuation model

Signal propagation model: lognormal fading

\[ P_i = H_i \frac{kP}{d_i^\alpha} \quad H_i = e^{W_i}; \text{lognormal r.v.} \]

\[ \ln P_i = \ln(kP) + \ln(d_i^{-\alpha}) + W_i \quad W_i \sim \mathcal{N}(0, \sigma^2) \]

- \( P = \text{transmission power} \)
- \( P_i = \text{received power at monitor } i \)
- \( d_i = \text{distance between the transmitter and monitor } i \)
- \( H_i = \text{lognormal random variable} \)

\( H_i \) – unknown to the monitor
- \( \text{represents the aggregated effect of randomness in the environment;}\)
- \( \text{eg: multi-path fading} \)
Stochastic attenuation model

Let \( z_i = \ln(p_i) \); where \( p_i \) is received power

We are given \((z_i, x_i, y_i)\) for \( i = 1, \ldots, N \)

Let \( Z = \ln(P) \), and \( \hat{\theta} = (Z, x, y) \)

The joint probability density function

\[
f(z; \hat{\theta}) = \prod_{i=1}^{N} f_{W_i}(z_i + \ln(d_i^e/k) - Z) = \prod_{i=1}^{N} \frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{(z_i + \ln(d_i^e/k) - Z)^2}{2\sigma^2}}
\]

Maximum Likelihood Estimate

**ML estimate** \((Z^*, x^*, y^*)\) is the value that maximizes the joint probability density function

\[
(Z^*, x^*, y^*) = \arg \max_{\hat{\theta}} f(z; \hat{\theta})
\]
ML estimate under stochastic model

\[
P^* = \frac{1}{k} \left( \prod_{i=1}^{N} p_i(d_i^*)^\alpha \right)^{1/N}
\]

\[
(x^*, y^*) = \arg \min_{(x,y)} \sum_{i=1}^{N} \left( \ln(p_i d_i^\alpha) - \frac{\sum_{j=1}^{N} \ln(p_j d_j^\alpha)}{N} \right)^2
\]

\[
d_i = \sqrt{(x_i - x)^2 + (y_i - y)^2} \quad \text{distance between some location (x,y) and monitor i}
\]

\[
d_i^* = \sqrt{(x_i - x^*)^2 + (y_i - y^*)^2} \quad \text{distance between estimated Tx. location (x,y) and monitor i}
\]

- \((x^*, y^*)\) is the solution to the minimization above, where the objective function
  is sample variance of \( \{\ln(p_i d_i^\alpha)\} \)
- \(P^*\) is proportional to the geometric mean of \( \{p_i(d_i^*)^\alpha\} \)
Asymptotic Optimality of ML estimate

Theorem IV. (Asymptotic optimality of ML estimate) Consider the random monitor placement scenario over a bounded region $\Gamma$. Let the monitor location $(x_i, y_i), \forall i$, be drawn independently from the distribution $F_{XY}(x, y)$. Let $F_{XY}(x, y)$ be such that,

1. $F_{XY}(x, y)$ is not a distribution over an arc of a circle or a straight line in $\Gamma$.

2. $E[|L(\hat{x}, \hat{y})|] < \infty$ and $\text{Var}(L(\hat{x}, \hat{y})) < \infty$, for all $(\hat{x}, \hat{y}) \in \Gamma$, where $L(\hat{x}, \hat{y}) \triangleq \frac{\alpha}{2} \ln \left( \frac{(x-\hat{x})^2 + (y-\hat{y})^2}{x^2 + y^2} \right)$.

Let $P_N^*$ denote the ML estimate of transmit power $P$ given by (23) for an $N$ monitor scenario. Then, almost surely,

$$\lim_{N \to \infty} P_N^* = P$$
Performance Evaluation

- **Synthetic data set**
  - $N = 2$ to 20 placed uniformly at random in a disk of radius $R$.
  - Received power is generated by i.i.d. lognormal fading model for each monitor.
  - Performance measured: averaged over estimation for 1000 transmitters.

- **Empirical data set**
  - Sensor network measurement data at U of Michigan.
  - Total 44 sensor devices. Received powers are measured between all pairs of devices.
    \[ \alpha = 2.3, \text{ and } \sigma_{dB} = 3.92. \]
  - Randomly choose $N=3,4,\ldots,10$ monitors out of 44 devices.

- **Estimators**
  - **MLE-Coop-fmin**: MLE with fminsearch for location estimation.
  - **MLE-Coop-grid**: MLE with location estimation among grid points.
  - **MLE-ideal**: MLE with known transmitter location.
  - **MLE-Pair**: Average of pair-wise MLEs.

- **Performance metric**
  \[
  E_K[\mathcal{E}_{dB}^2] = E_K \left[ (10 \log_{10}(P^*/P))^2 \right]
  \]
Evaluation

**Synthetic data set**

- MLE-Coop-fmin
- MLE-Coop-grid
- MLE-Pair
- MLE-ideal

**Empirical data set**
(MLE-Coop-grid)
Multi-transmitter estimation

\[ \hat{P}_j = \sum_{i=1}^{K} \frac{P_i}{d_{i,j}^\alpha}, \quad \text{for } j = 1, \ldots, N \]

- \( K \): # of transmitters
- \( N \): # of monitors
- \( d_{i,j} \): distance between Tx I and monitor j

**Question:**
- How many transmitters are out there (at least)?
- What are their transmission powers?
- How many monitors do we need?

\[ \theta^* = \arg\min_{\theta \in \Omega} \sum_{j=1}^{n} \left( \hat{P}_j - \sum_{i=1}^{k} \frac{P_i}{d_{i,j}^\alpha} \right)^2, \]

s.t. \( P_i \geq 0, \quad i = 1, \ldots, k. \)
Conclusion and Open Problems

- **Blind estimation of transmission power**
  - Studied estimators for deterministic and stochastic signal propagation
  - Utilized spatial diversity in measurements
  - Obtained asymptotically optimal ML estimate
  - Presented numerical results quantifying the performance

- **Open problems**
  - Non-trusted monitoring scenario
  - Monitoring under heterogeneous channel characteristics

Reference:
Thank you.