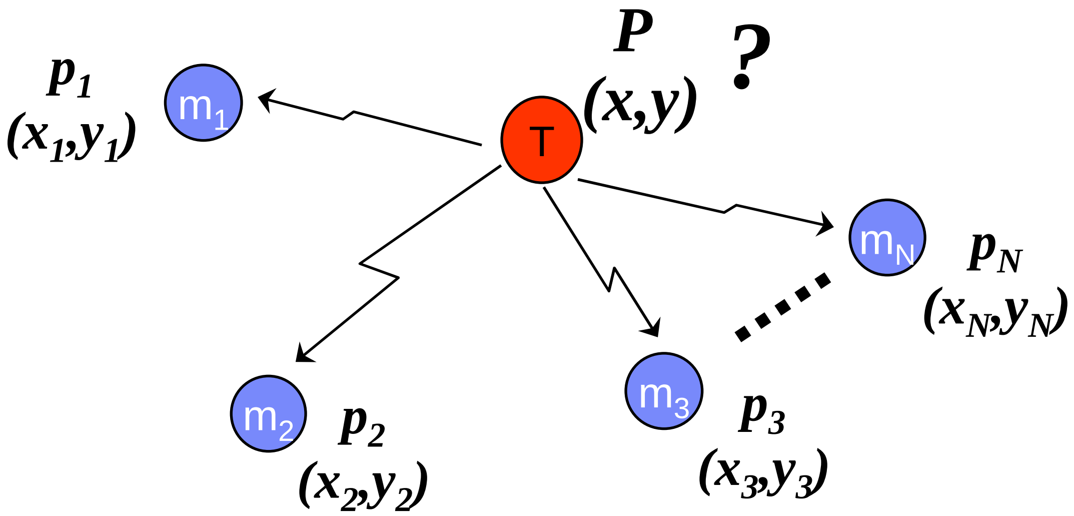
Blind Estimation of Transmit Power in Wireless Networks

Murtaza Zafer (IBM Research), Bongjun Ko (IBM Research), Chatschik Bisdikian (IBM Research) and Ivan Ho (Imperial College, UK)

Transmit-power Estimation: Problem Synopsis



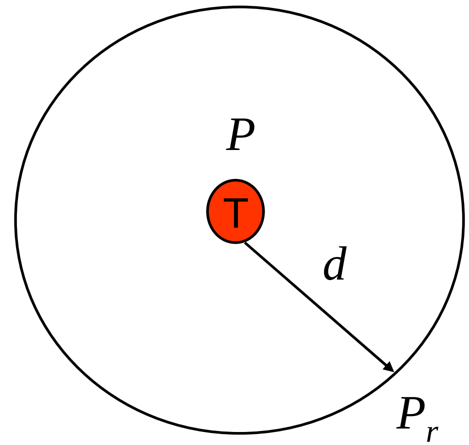
- Node T is a wireless transmitter with Tx power = *P* (unknown), at a position (x, y) (unknown)
- Nodes $\{m_1,\ldots, m_N\}$ are monitors that measure **received power** $\{p_i\}$
- ***Goal*** – given $\{p_i\}$ and $\{(x_i,y_i)\}$ (monitor locations), estimate unknown *P* and (*x*, *y*).
- ***Difficulty:*** "Blind" estimation – no prior knowledge (statistical or otherwise) of the location or transmit power of the transmitter.

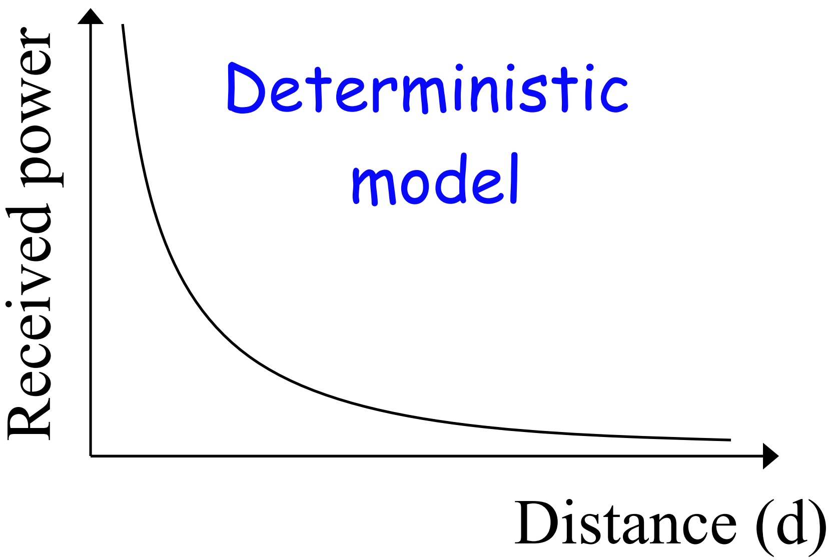
Motivation

- Applications
  - Signal jamming attack detection in MANET.
  - Node mis-configuration detection.
  - Primary user detection in cognitive radio networks.
  - Event intensity detection in sensor network.
  - Power-aware radio resource control with unknown transmit power.
  - Location identification of wireless users.

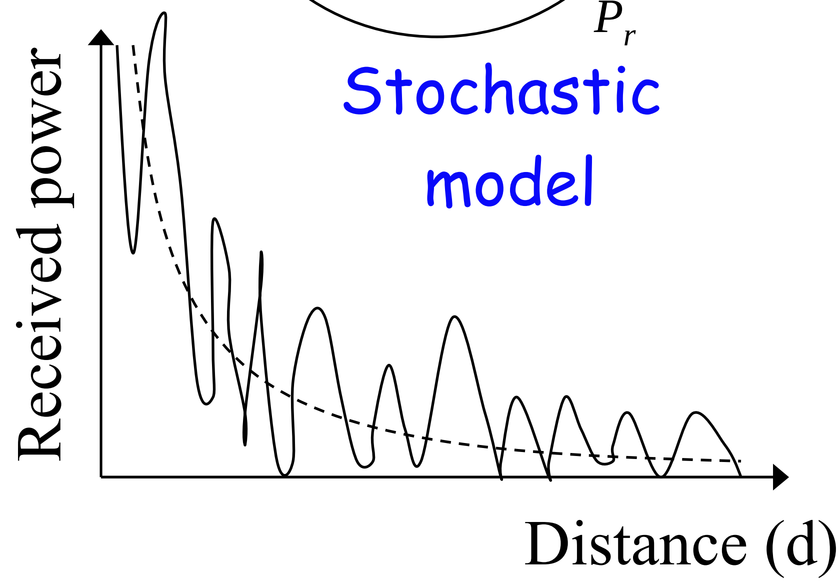
Wireless signal attenuation

P = transmission power
P_r = received power
d = distance between the transmitter and receiver
α = attenuation factor, ($\alpha > 1$)
k = normalizing constant





$$P_r = kP / d^{\alpha}$$



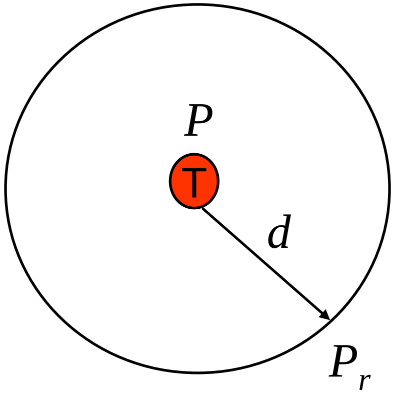
$$P_i = H_i \, kP / d_i^{\alpha}$$

$$H_i = e^{W_i} \text{ ; lognormal r.v.}$$

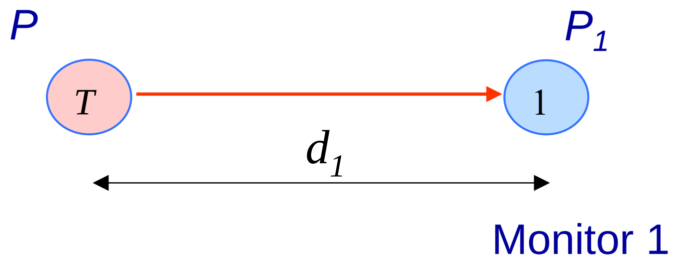
Estimation under deterministic model

Deterministic propagation model:

$$P_r = kP / d^{\alpha}$$



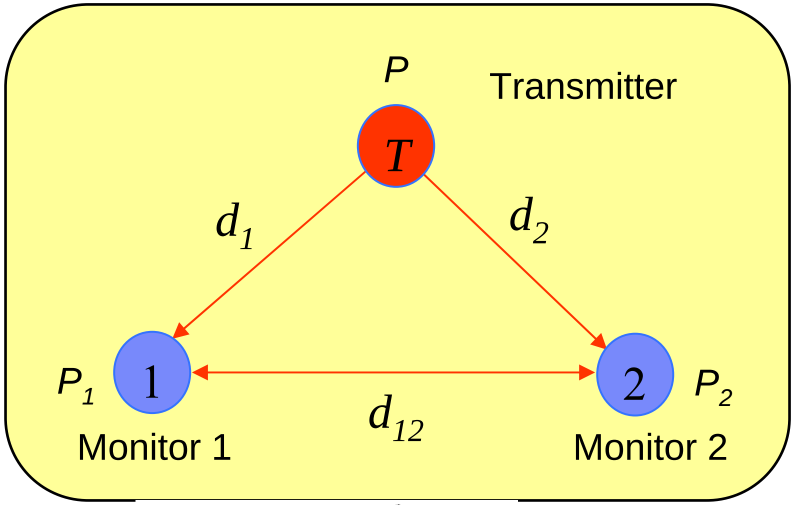
Single monitor measurement



best estimate of transmit power:

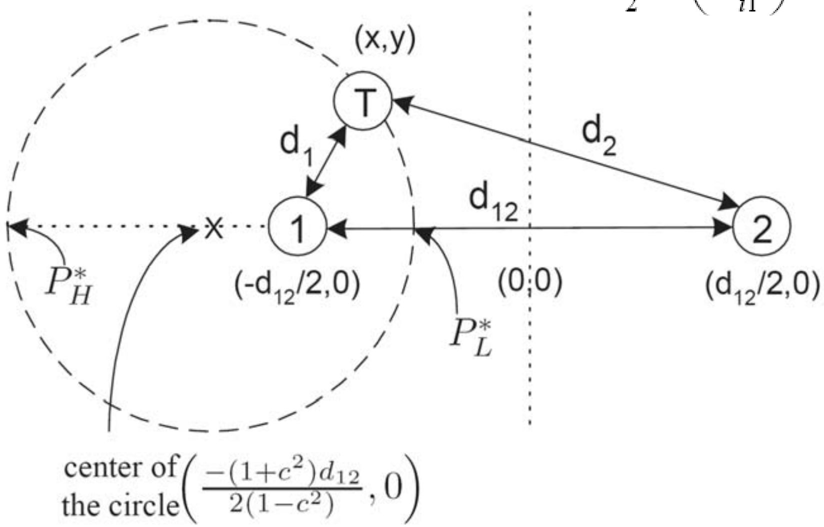
$$P^* = P_1$$

Two monitors



$$P_i = kP / d_i^{\alpha}$$

Locus of transmitter : using $\dfrac{d_1}{d_2} = \left(\dfrac{P_{i2}}{P_{i1}}\right)^{\frac{1}{\alpha}} = c_1$



Lower bound of *P**:

By the triangular inequality: $d_1 + d_2 \geq d_{12}$

$$P_i^* = \left(\frac{d_{12}}{(k/P_{i1})^{\frac{1}{\alpha}} + (k/P_{i2})^{\frac{1}{\alpha}}} \right)^{\alpha}$$

Multiple monitors

Multiple monitor scenario

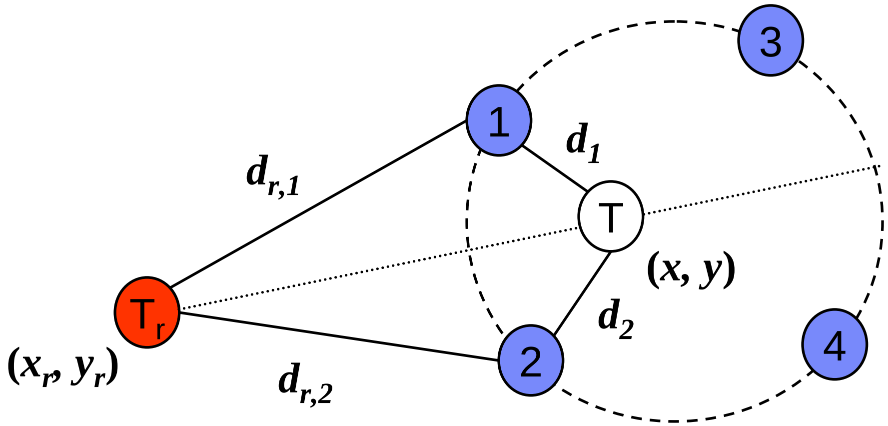
$$\frac{d_1}{d_2} = \left(\frac{P_2}{P_1}\right)^{1/\alpha} (= c_1); \quad \frac{d_2}{d_3} = \left(\frac{P_3}{P_2}\right)^{1/\alpha} (= c_2); \quad ----- \quad \frac{d_{N-1}}{d_N} = \left(\frac{P_N}{P_{N-1}}\right)^{1/\alpha} (= c_{N-1})$$

- With multiple monitors – diversity in measurements
- System of equations with unknowns *(x,y,P)*
- *We should be able to solve these equations to obtain exact P ?*

Answer: Yes and No !!

Multi-monitor estimation under deterministic model

Theorem I: *There is a unique solution (P^*, x^*, y^*)* ***except when*** *the monitors are placed on an arc of a circle.*



Proof:

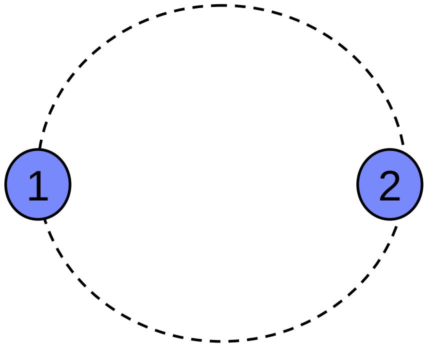
- A location (x, y) is a solution if and only if it satisfies $d_1/d_2=c_1, \ldots, d_{N-1}/d_N = c_{N-1}$
- The actual location (x_r, y_r) is one solution; thus $d_{r,1}/d_{r,2}=c_1, \ldots, d_{r,N-1}/d_{r,N} = c_{N-1}$
- There exists another solution at (x, y) if and only if, $d_{r,1}/d_{r,2} = d_1/d_2, \ldots$; equivalently,

$$\frac{d_i}{d_{r,i}} = \beta, \ \text{(a constant)}, \ \forall i = 1, \ldots, N$$

Deterministic model

Multiple monitor scenario

Corollary 1: *Two monitors always has multiple solutions*

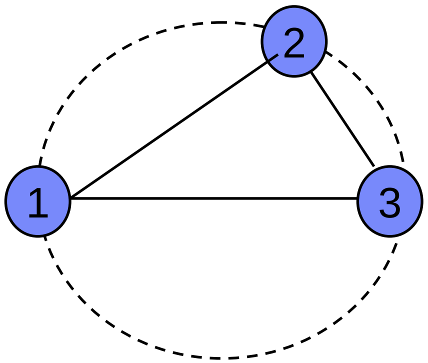


Deterministic model

Multiple monitor scenario

Corollary 1: *Two monitors always has multiple solutions*

Corollary 2: *Three monitors always has multiple solutions*



In general, for any regular polygon placement of monitors the transmission power cannot be uniquely determined !

For all non-circular placement of monitors, transmission power can be uniquely determined.

Stochastic attenuation model

Signal propagation model: *lognormal fading*

$$P_i = H_i \, kP / d_i^{\alpha} \qquad H_i = e^{W_i} \text{ ; lognormal r.v.}$$

$$\ln P_i = \ln(kP) + \ln(d_i^{-\alpha}) + W_i \qquad W_i \sim \mathcal{N}(0, \sigma^2)$$

P = transmission power
P_i = received power at monitor i
d_i = distance between the transmitter and monitor i
H_i = lognormal random variable

H_i – *unknown to the monitor*
– *represents the aggregated effect of randomness in the environment; eg: multi-path fading*

Stochastic attenuation model

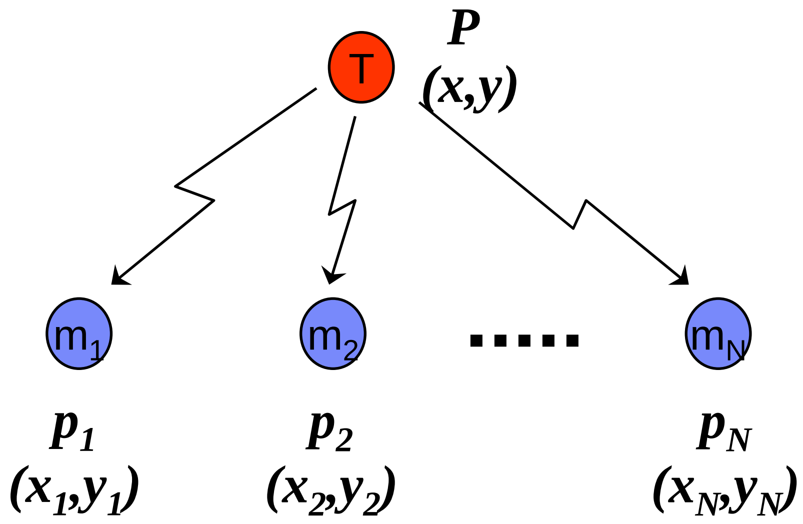
Let $z_i = \ln(p_i)$; where p_i is received power

We are given (z_i, x_i, y_i) for $i = 1, .., N$

Let $Z = \ln(P)$, and $\hat{\theta} = (Z, x, y)$

The joint probability density function

$$f(\mathbf{z}; \hat{\theta}) = \prod_{i=1}^{N} f_{W_i}(z_i + \ln(d_i^{\alpha}/k) - Z)$$
$$= \prod_{i=1}^{N} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(z_i + \ln(d_i^{\alpha}/k) - Z)^2}{2\sigma^2}}$$



Maximum Likelihood Estimate

ML estimate *(Z*,x*,y*)* is the value that maximizes the joint probability density function

$$(Z^*, x^*, y^*) = \arg\max_{\hat{\theta}} f(z; \hat{\theta})$$

ML estimate under stochastic model

$$P^* = \frac{1}{k}\left(\prod_{i=1}^{N} p_i (d_i^*)^\alpha\right)^{1/N}$$

$$(x^*, y^*) = \arg\min_{(x,y)} \sum_{i=1}^{N}\left(\ln(p_i d_i^\alpha) - \frac{\sum_{j=1}^{N}\ln(p_j d_j^\alpha)}{N}\right)^2$$

$d_i = \sqrt{(x_i - x)^2 + (y_i - y)^2}$ *distance between some location (x,y) and monitor i*

$d_i^* = \sqrt{(x_i - x^*)^2 + (y_i - y^*)^2}$ *distance between estimated Tx. location (x,y) and monitor i*

- *(x*,y*)* is the solution to the minimization above, where the objective function is sample variance of $\{ln(p_i d_i^{\alpha})\}$
- *P** is proportional to the geometric mean of $\{p_i(d^*_i)^{\alpha}\}$

Asymptotic Optimality of ML estimate

Theorem IV. (Asymptotic optimality of ML estimate) *Consider the random monitor placement scenario over a bounded region* Γ*. Let the monitor location* $(x_i, y_i), \forall i$*, be drawn independently from the distribution* $F_{XY}(x,y)$*. Let* $F_{XY}(x,y)$ *be such that,*

1. $F_{XY}(x,y)$ *is not a distribution over an arc of a circle or a straight line in* Γ*.*

2. $E[|L(\hat{x},\hat{y})|] < \infty$ *and* $Var(L(\hat{x},\hat{y})) < \infty$*, for all* $(\hat{x},\hat{y}) \in \Gamma$*, where* $L(\hat{x},\hat{y}) \triangleq \frac{\alpha}{2} \ln\left(\frac{(x-\hat{x})^2+(y-\hat{y})^2}{x^2+y^2}\right)$.

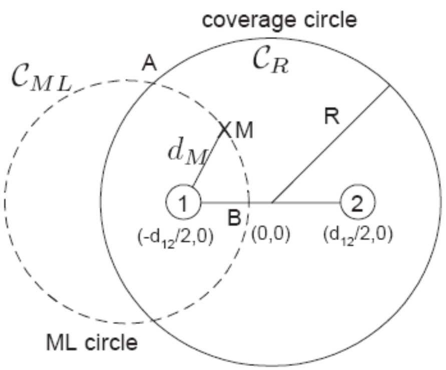
Let P_N^* *denote the ML estimate of transmit power* P *given by (23) for an* N *monitor scenario. Then, almost surely,*

$$\lim_{N\to\infty} P_N^* = P$$

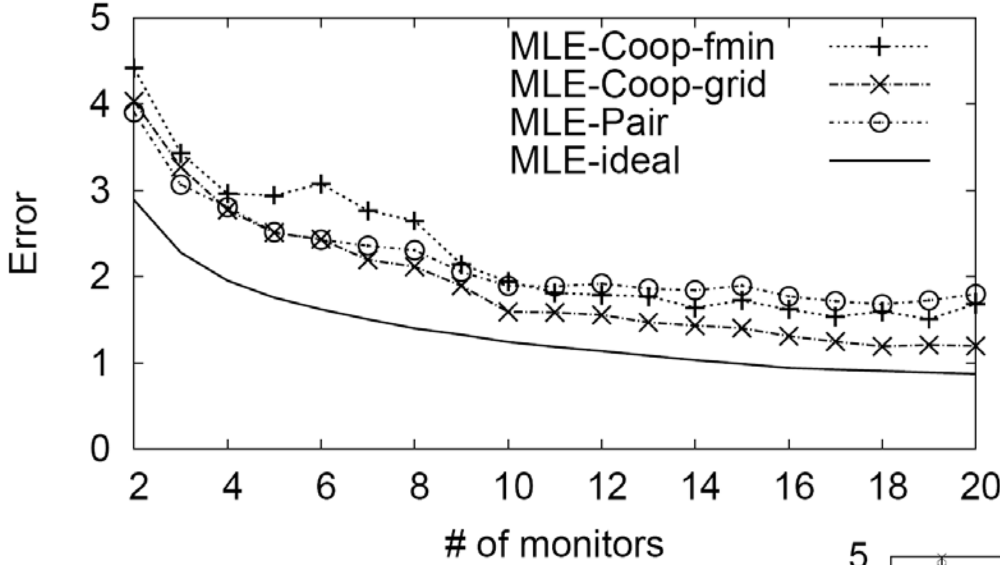
Performance Evaluation

- **Synthetic data set**
  - N = 2 to 20 placed uniformly at random in a disk of radius R.
  - Received power is generated by i.i.d. lognormal fading model for each monitor.
  - Performance measured: averaged over estimation for 1000 transmitters.
- **Empirical data set**
  - Sensor network measurement data at U of Michigan.
  - Total 44 sensor devices. Received powers are measured between all pairs of devices. $\alpha = 2.3$, and $\sigma_{dB} = 3.92$.
  - Randomly choose N=3,4,…,10 monitors out of 44 devices.
- **Estimators**
  - **MLE-Coop-fmin** : MLE with fminsearch for location estimation.
  - **MLE-Coop-grid**: MLE with location estimation among grid points.
  - **MLE-ideal**: MLE with known transmitter location.
  - **MLE-Pair**: Average of pair-wise MLEs.
- **Performance metric**

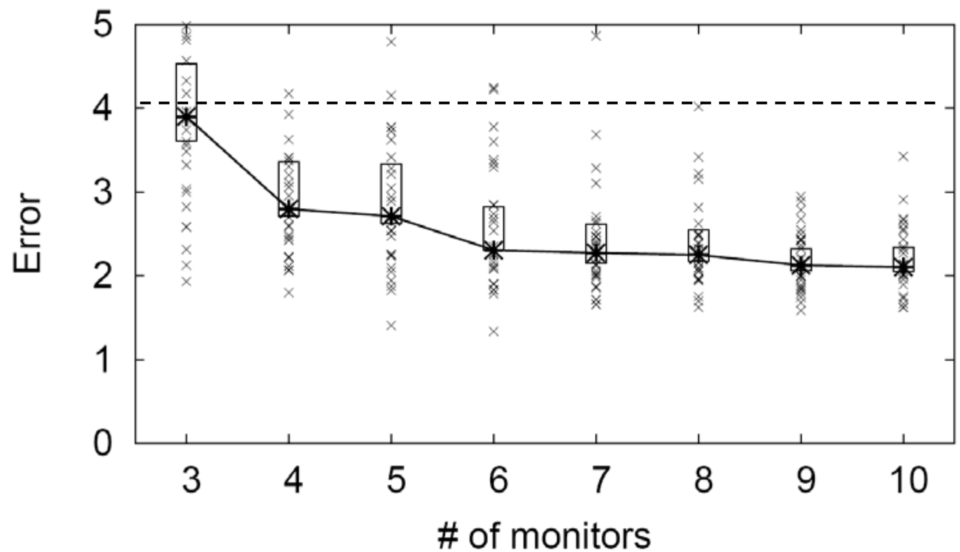
$$\mathbf{E}_K[\mathcal{E}_{dB}^2] = \mathbf{E}_K\left[(10\log_{10}(P^*/P))^2\right]$$



Evaluation



Synthetic data set



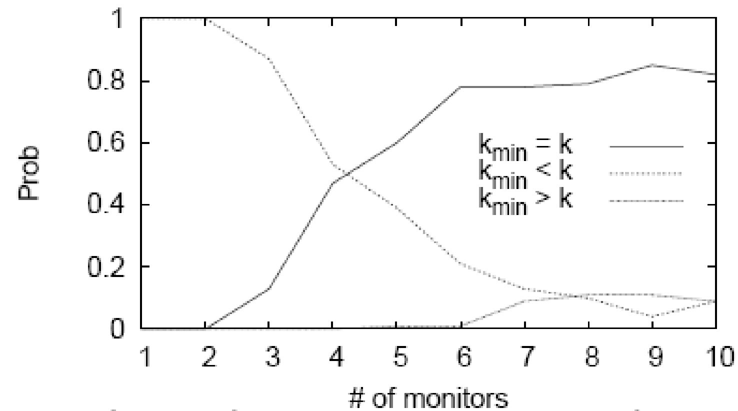
Empirical data set

(MLE-Coop-grid)

Multi-transmitter estimation

$$\hat{P}_j = \sum_{i=1}^{K} \frac{P_i}{d_{i,j}^{\alpha}}, \quad \text{for } j = 1, \cdots, N$$

- K: # of transmitters
- N: # of monitors
- $d_{i,j}$: distance between Tx I and monitor j



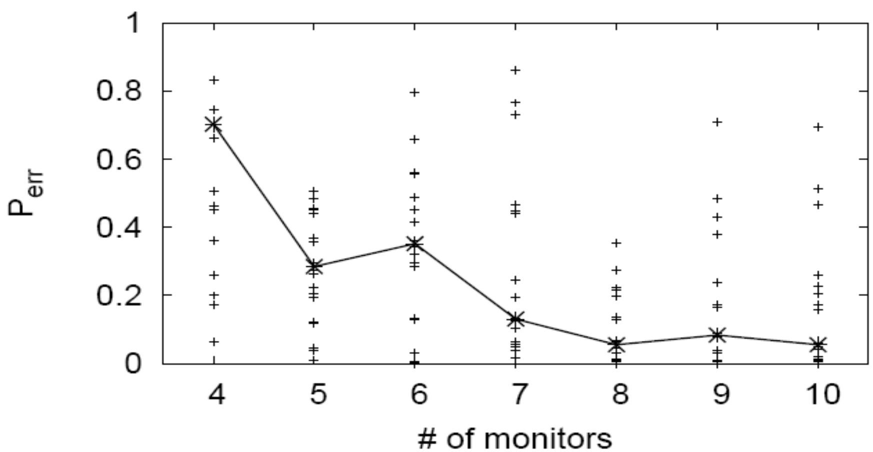
(a) Prob{k_{min}=K} (correct estimation), Prob{k_{min}¡K} (under-estimation), and Prob{k_{min}¿K} (over-estimation) with R=100

Question:

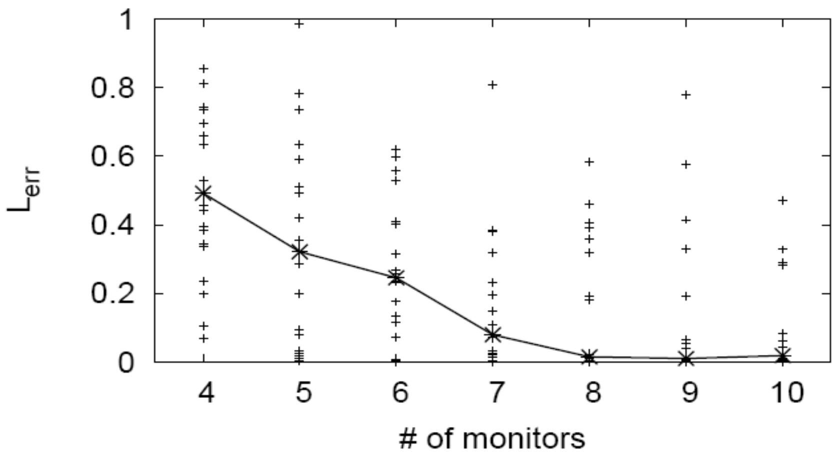
- How many transmitters are out there (at least)?
- What are their transmission powers?
- How many monitors do we need?

$$\theta^* = \arg\min_{\theta \in \Omega} \sum_{j=1}^{n} \left(\hat{P}_j - \sum_{i=1}^{k} \frac{P_i}{d_{i,j}^{\alpha}} \right)^2,$$

$$\text{s.t.} \quad P_i \geq 0, \quad i = 1, \cdots, k.$$



(a) P_{err}



(b) L_{err}

Conclusion and Open Problems

- ***Blind estimation of transmission power***
  - Studied estimators for deterministic and stochastic signal propagation
  - Utilized spatial diversity in measurements
  - Obtained asymptotically optimal ML estimate
  - Presented numerical results quantifying the performance
- ***Open problems***
  - Non-trusted monitoring scenario
  - Monitoring under heterogeneous channel characteristics

Reference:

• I. W. Ho, B. Ko, M. Zafer, C. Bisdikian, and K. Leung, "*Cooperative Transmit-Power Estimation in MANETs*", WCNC 2008.

• M. Zafer, B. Ko and I. W. Ho, "*Cooperative transmit-power estimation under wireless fading*", *ACM Mobihoc 2008.*

• I. W. Ho, B. Ko, and M. Zafer, "*Blind Estimation of Transmit-Power for Multiple Wireless Sources*", MILCOM 2008.

Thank you.